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Product Analysis: Libor/Euribor-in-Arrears (LIA)

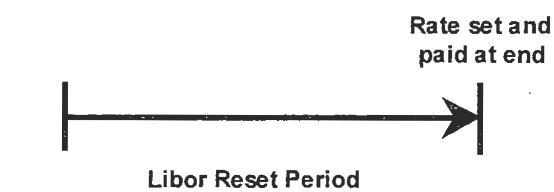
Description

LIA Swap Structure

A structure often used when yield curves are steep is Libor (or Euribor) in arrears.

The normal process of making payments in Libor **fixes** the rate at the beginning of the period and **pays** the rate at the end:

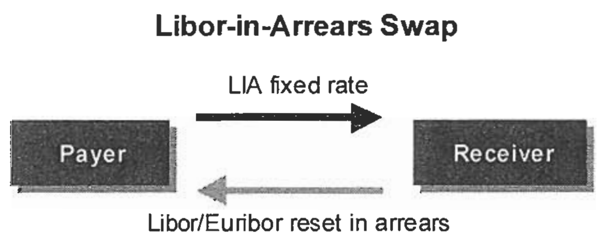
Libor-in-arrears payments change this pattern. Libor is **fixed at the end** of the period and **paid at the end**, two days later:



Normally, Libor is set using the rate for the same length period beginning at the end of the relevant reset period.

For example, if we are using 6-month Libor resets, then we will set the rate for the period just ending using the market level of 6-month Libor for the period just beginning. This is not the only possible approach, however. We might also choose to use the market rate for a longer or shorter Libor period, for example.

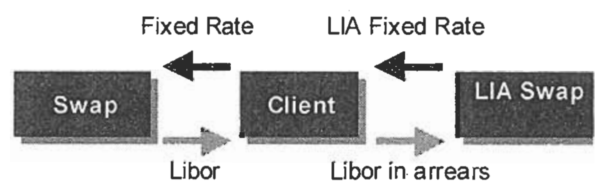
The basic LIA swap is very much like a standard interest rate swap:



The LIA fixed rate is more or less equal to the rate on a forward starting swap of the same tenor beginning one Libor reset period forward.

In an **upward-sloping** yield curve, the LIA fixed rate will be **higher** than the normal swap rate. In a **downward-sloping** yield curve, the LIA fixed rate will be **lower** than the normal swap rate.

The LIA swap can be combined with a regular interest rate swap to exchange Libor in arrears against Libor.



In an upward-sloping yield curve, the LIA fixed rate will be higher than the normal swap rate by a fixed amount.

If the 3-year LIA swap rate was 4.75% and the 3-year standard swap rate was 4.50%, the client would receive a spread equivalent to the difference of 0.25% each period.

This would produce the following position for the client:

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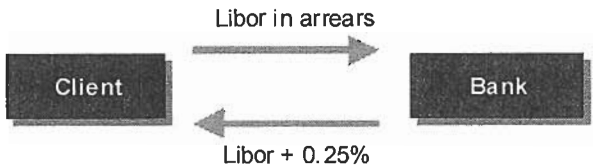
Libor-in-Arrears Swap



As long as Libor did not rise by more than 0.25% on average over each reset period for the next 3 years, the client would come out ahead.

His view would have to be that the forward curve "forecasts" Libor rising faster and earlier than the client believes would happen. In his view, the curve is too steeply upward sloping.

Related Products

Libor in arrears uses a market rate as an index or reference to determine a payment amount each reset period. The market rate it uses, however, Libor set in arrears, is not the actual market rate for the period to which it will be applied. In this case, it is the "natural market rate" for the subsequent period.

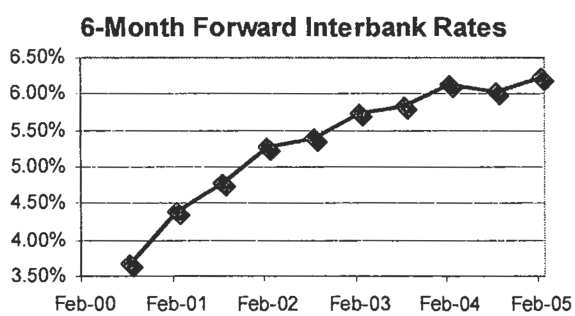
In this respect Libor in arrears is similar to a constant maturity swap or bond. The difference is only a question of which reference rate we choose. If the reference rate is a short-term deposit rate like Libor or Euribor, we call the structure *Libor in arrears.* If the reference rate is a longer-term rate such as a swap rate, we call the structure a *constant maturity swap*. If the reference rate is a government bond yield we call it the constant maturity Treasury (or *Bund*, *OAT*, Gilt, etc.).

As we will discuss in the section called ***Valuation Logic***, using a market rate as a reference to establish payments in periods for which it is not the "natural" market rate requires an adjustment to the market rates we use to price the structure. Technically, this is known as changing the numeraire," and the market refers to the necessary adjustment as a *convexity adjustment*. We will discuss the convexity adjustment for Libor in arrears in the ***Valuation Logic*** section following.

Recent Market Example

The Euro yield curve is fairly steep in February 2000. This is fertile ground for considering Libor in arrears.

Chart 1



Most of the steep rise in forward 6-month Euribor rates occurs during the first three to four years through February 2004. Market rates reflect this steepness:

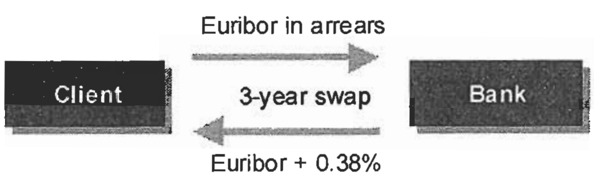
Table 1

Deposits	Euribor	6-mo Forwards
O/N	3.30%	
1M Libor	3.31%	
3M Libor	3.49%	
6M Libor	3.68%	3.68%
12M Libor	4.07%	4.37%
Basis	A/360	
Par Swaps	**€ Swaps**	**6-mo Forwards**
2 Year	4.64%	5.28%
3 Year	4.98%	5.74%
4 Year	5.25%	6.13%
5 Year	5.44%	6.22%
6 Year	5.59%	6.38%
7 Year	5.73%	6.65%
8 Year	5.85%	6.79%
9 Year	5.93%	6.62%
10 Year	5.99%	6.58%
Basis	30/360 p.a.	

A 3-year Euribor-in-arrears swap would be priced at the following level:

Pay	Euribor in arrears
Receive	Euribor + 0.38%

Euribor-in-Arrears Swap



This position would make sense for a client paying Euribor on bank debt, for example. It would support a view by the client that Euribor will not rise as steeply as the forward curve implies. In fact, the spread of 0.38% implies an average rise in 6-month Euribor of 0.38% every six months for the next three years.

As will see later in the ***Price Sensitivities*** section, the risk taken on by the client in the picture above would be that Euribor rises faster than 0.38% every six months ***on average***. If this does not happen he will come out ahead.

Favorable Market Environment

Libor in arrears is well suited to steep yield curve environments, particularly when the steepness is due to long term rates sitting at "comfortable" historical levels while short-term rates are at cyclical lows. In this environment, many market observers and users will not believe that years of sustained short-term rate increases are on offer. While they can often imagine one or two central bank short-term rate increases, they find it difficult to believe that steady rises every few months will follow for the next three to five years. This is a breeding ground for yield curve plays, and LIA is prominent among them—and probably the simplest and least risky. The Euro curve in early 2000 is such a curve, although its steepness is relatively mild and LIA spreads are not dramatic.

A good rule of thumb for the spreads on LIA swaps is to look at the spread to the spot swap for a swap of the same tenor shifted one Libor reset period forward. For a 3-year LIA swap we can compare the 3-year swap to the 3-year swap 6 months forward. For a 4-year LIA swap, we can compare the 4-year swap to the 4-year swap 6 months forward. The table below summarizes these comparisons in February 2000:

Table 2

Tenor	Spot Swaps	6 mo. Forward	Difference
2y	4.64%	5.07%	0.43%
3y	4.98%	5.35%	0.37%
4y	5.25%	5.56%	0.31%
5y	5.44%	5.71%	0.27%
6y	5.59%	5.84%	0.25%
7y	5.73%	5.96%	0.23%
8y	5.85%	6.05%	0.20%
9y	5.93%	6.11%	0.18%
10y	5.99%	6.16%	0.17%

The spread we showed above for the 3-year LIA was 0.38%. The rule of thumb spread is 0.37%.

The steeper the curve, the greater the difference between a spot starting swap rate and its 3-, 6- or 12-month forward brother. When spreads move above 0.35% people start to pay attention to LIA. Above 0.50%, LIA appears compelling.

Price Sensitivities

In this section we examine the sensitivity of the mark-to-market value of an in-arrears swap to changes in market rates. We approach changing market rates in three ways: 1) Parallel shifts of the entire yield curve; 2) One basis point shifts of each market rate used in establishing the market yield curve; and 3) Non-parallel yield curve shifts of varying amounts.

Value of a Basis Point

Value of a basis point, also called VBP, BPV, PVBP, PV01, and dollar duration, refers to the average

amount by which the mark-to-market value (MMV) of any instrument changes when the yield curve is shifted up and down by 0.01%. It is an ***absolute measure*** of pure price change.

Modified duration is a ***relative measure*** of change in the mark-to-market value of an instrument or structure. It measures the average amount of MMV change when the yield curve is shifted up and down by 0.01%. Modified duration is the PV01 divided by the initial MMV. For a par bond or swap, the modified duration is equal to the PV01.

The 0.01% shifts used for PV01 and modified duration are assumed to take place across the entire yield curve. They are thus known as ***parallel yield curve shifts***. PV01 does not capture the risk of change in the shape of the yield curve. This is an important point, as shape changes are likely to cause the most damage to structured interest rate derivatives.

Factor Sensitivity

Factor sensitivity, also called key rate duration, is a measure of the PV01 of any instrument associated to a 0.01% change in ***each market rate*** used to establish the pricing curve. While there is only one measure of PV01 for a given instrument, it has as many ***factor sensitivities*** as there are market-input rates.

In the examples outlined in this paper, we will use a consistent set of 17 market-input rates, beginning with overnight (O/N), 1-week, 1-, 3-, 6- and 12-month deposit rates, and 2-, 3-, 4-, 5-, 6-, 7-, 8-, 9-, 10-, 12-, 15- and 20-year swap rates.

In each case, we will shift a single market rate up and down by 0.01%, calculate the average change in the instrument's MMV, put the market rate back to its initial value, and move to the next market input rate. Each instrument will therefore show up to 17 different factor sensitivities.

In practice it is more common to use deposit futures, because traders will use futures rather than deposit rates to construct their hedges. For illustrative purposes, however, deposit rates are equally good and more easily understood by the larger number of people in the market.

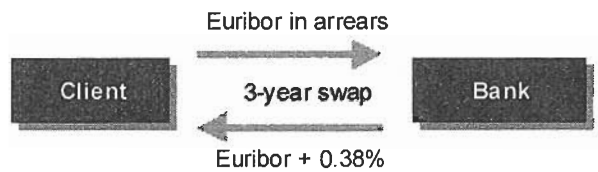
Non-Parallel Yield Curve Shifts

There is no standard market convention for measuring an instrument's price sensitivity to non-parallel shifts of the yield curve. In the examples outlined in this paper we will move the curve in three ways: steeper, higher and flatter, and lower and flatter. The amounts by which a given curve is adjusted are selected depending on the currency and the level of its curve.

In-Arrears Swap Sensitivities

If he is willing a client might take the following position for 3 years:

Euribor-in-Arrears Swap



As long as Libor does not go up by more than 0.38% every six months for the next three years, the client will be better off. This is a trade that benefits from a flattening of the curve.

Modified Duration and Convexity

By agreeing to lock in the level of LIA against regular Libor at 0.38%, the client is betting that the curve will flatten faster than the forward curve implies it will.

Only if the curve steepens by more than 0.38% per period on average will this position go against the client. He receives the spread fixed at a level of 0.38% and pays the spread between each Libor fixing and the following fixing at future market rates. If the rise of Libor from reset date to reset date is less than 0.38% he receives money from the swap.

Every structure that offsets one floating rate against another will show very little PV01 or modified duration. This position appears to carry very little risk:

Table 3

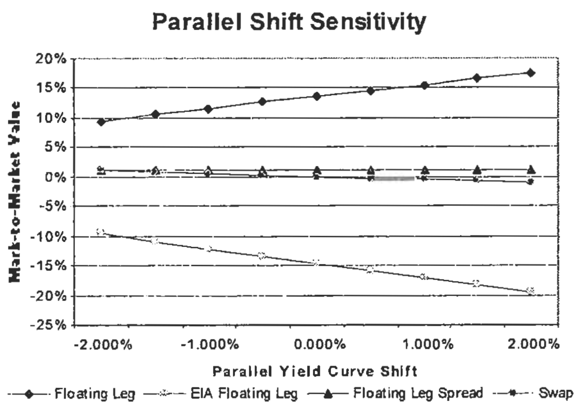
Curve Shift	Euribor	EIA	Swap
MMV			
Curve ↑ 0.01%	13.66%	-13.67%	-0.0050%
Initial MMV	13.64%	-13.64%	0.0000%
Curve ↓ 0.01%	13.62%	-13.62%	0.0050%
Change in MMV			
Curve ↑ 0.01%	0.0198%	-0.0248%	-0.0050%
Initial MMV	0.0000%	0.0000%	0.0000%
Curve ↓ 0.01%	-0.0199%	0.0248%	0.0050%
Modified Duration			0.4988

Because the floating rates reset each period to the market rate, the price risk of this structure is very low,

only about a half-year. This is the length of the reset periods.

The VBP of this swap, at €4,988 against notional principal of €100 million, is tiny. Even this small amount results only because the first Euribor fixing is fixed, and reduces the sensitivity of the remaining floating-rate resets on the Euribor leg to a small degree. The Euribor leg thus shows slightly less sensitivity to rate shifts than the EIA leg. Basically, as shown in the chart below, the two legs offset each other and the result is very little parallel curve shift risk.

Chart 2



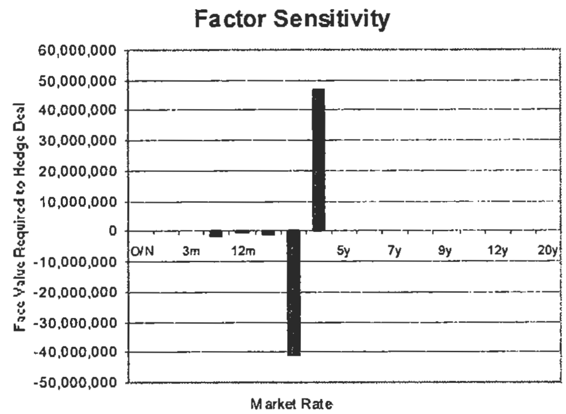
This picture is an illusion, however, and should not be taken to mean that there is no interest rate risk in this structure. There is.

The interest rate risk of LIA is not about parallel shifts of the yield curve, which are the only type of rate shift that PV01 and modified duration can imagine. Rather the risk of LIA is precisely to ***non-parallel shifts*** of the yield curve. That is what the structure targets for its performance.

Factor Sensitivity

Factor sensitivity for a LIA swap shows the trade for what it really is: a position in offsetting series of Libor resets, one beginning now and one beginning in six months (or three months or 12 months). This can be replicated fairly closely by trading two spot-starting swaps against each other. In the case of the deal we showed above, a 3-year EIA swap based on 6-month Euribor, the replicating trade is a 3-year swap against a 4-year swap. It would be a 3-year swap against a 3½-year swap except that there is no market rate for a 3½-year swap.

Chart 3



The chart above shows the swaps a client could use to replicate his position. He would need to pay fixed on roughly €40 million at 3 years and receive fixed on roughly €45 million at 4 years. This is a spread trade targeting the shape of the yield curve at three years.

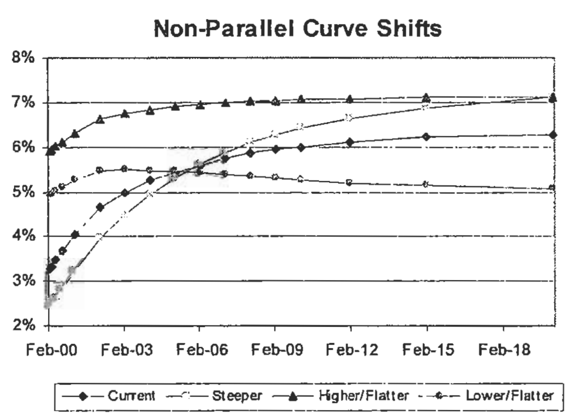
If he pays fixed at 3 years and receives fixed at 4 years he will benefit if the 3-year rate rises relative to the 4-year rate, or if the yield curve flattens (actually, flattens faster than the yield curve implies). This is identical to the client position we described above.

The two swaps shown replicate the risk position of the EIA swap. If the client were to do them the other way around, i.e. pay fixed at 3 years and receive fixed at 4 years, he could hedge the EIA trade.

Non-Parallel Curve Shifts

The curves used to measure the swap's sensitivity to changes in the shape of the yield curve are shown below.

Chart 4



We can measure the change in the MMV of the swap should the curve shift to any of the curves in the chart.

Table 4

Curve Shift	Euribor	EIA	Swap
MMV			
Initial MMV	14.71%	-14.71%	0.0000%
Steeper	13.95%	-14.01%	-0.0637%
Higher/Flatter	17.66%	-18.24%	-0.5707%
Lower/Flatter	15.18%	-14.97%	0.2163%
Change in MMV			
Steeper	-0.7643%	0.7006%	-0.0637%
Higher/Flatter	2.9534%	-3.5241%	-0.5707%
Lower/Flatter	0.4719%	-0.2557%	0.2163%

The curve sensitivities in this swap are at 3 years and 4 years. The MMV changes for the various curves therefore depend first on what happens at this point in the yield curve. A second factor is what happens to the value of the first Euribor reset that has been fixed already.

When the curve steepens, the 3-year rate falls relative to the 4-year rate. This results in a small loss for the EIA payer and the swap MMV declines by 0.0637%. When the curve inside 5 years moves lower, the PV of both (floating-rate) legs falls. The steep curve means that the value of the EIA resets does not fall as much as the value of the Euribor resets. This results in a loss for the payer of EIA.

When the curve moves higher and flattens the 3- and 4-year rates both rise, but the 3-year rate rises relatively faster. This results in a big loss for the EIA payer. At first look, this does not make sense as the EIA payer was supposedly positioned to benefit from a flattening of the yield curve. The first Euribor fixing—received by the EIA payer—is the culprit here, as it is fixed at a level far below the new market rate.

When the curve moves lower and flattens, the position behaves as it is "supposed" to. The gain of 0.2163% is the result of a flatter forward curve, which is not lost to the negative effect of the first Euribor fixing, as the curve has not risen too high in the short end.

Historical Market Examples

October 1992

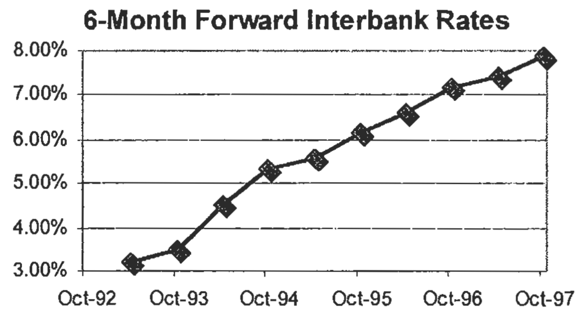
In October 1992 the US dollar yield curve was very steep. Swap rates from that time were as follows:

Table 5

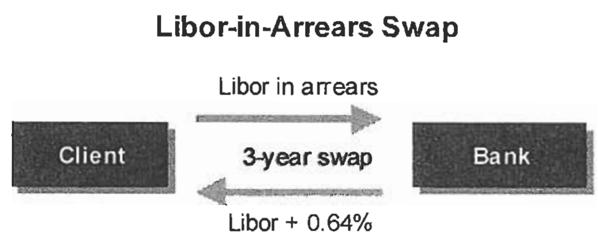
Deposits	Rate
3M Libor	3.00%
6M Libor	3.17%
12M Libor	3.33%
Par Swaps	**Rate**
2 Yr.	4.11%
3 Yr.	4.66%
4 Yr.	5.18%
5 Yr.	5.70%
6 Yr.	6.02%
7 Yr.	6.34%
8 Yr.	6.49%
9 Yr.	6.64%
10 Yr.	6.79%

A chart of 6-month USD Libor forward rates shows the steepness in the yield curve:

Chart 5



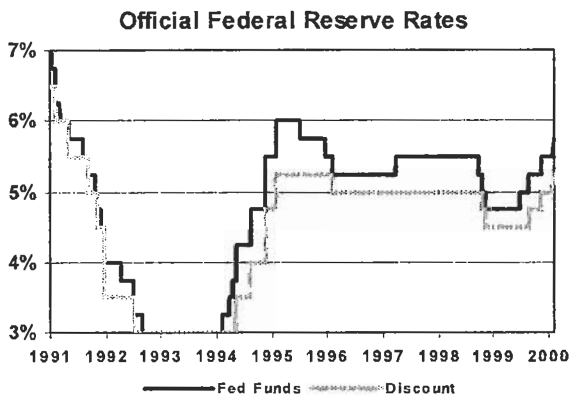
The spread on a 3-year LIA swap based on 6-month Libor starting 6 months forward (i.e. running from 12 April 1993 through 12 April 1996 but agreed on 12 October 1992) would have been some 0.64%.



Suppose a counterparty chose to receive this spread and pay Libor in arrears for the three years beginning April 1993. What would the performance of the trade have looked like?

We can evaluate the performance of the swap against historical market data. This is a particularly interesting period, as on 4 February 1994 the Federal Reserve Open Market Committee raised the Fed funds target rate in the U.S. for the first time in several years.

Chart 6



The client position in the swap as pictured is designed to benefit from a flattening of the yield curve—or from increases in 6-month Libor less than those implied in the forward curve. The spread of 0.64% indicates implied rises averaging 0.64% every six months for the three year period. How does this swap perform against one of the most dramatic periods of central bank tightening we have seen in years?

Two approaches are useful: 1) Mark-to-market value and 2) Cash flow paid to or from the swap each period. We will examine both of these at each reset date through the maturity of the swap.

April 1993

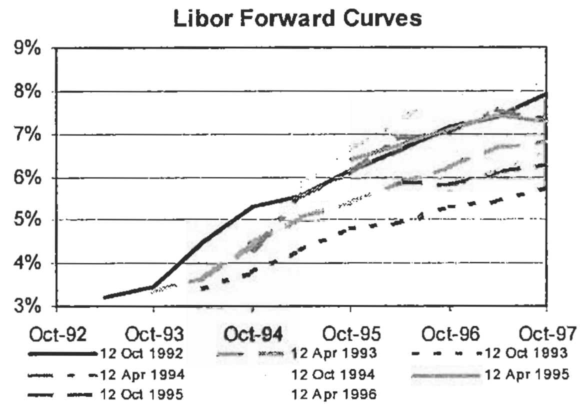
The first checkpoint is the forward start date, i.e. 12 April 1993.

Table 6

Swap Component	PV
Floating Leg	12.8524%
Floating Leg Spread	1.8021%
EIA Floating Leg	-14.2363%
Swap	0.4199%

The swap position is positive by 0.42% because the yield curve has flattened. The relevant forward curve starts above Oct-93 below, and can be compared to the curve that starts between Oct-92 and Oct-93, against which the spread of 0.64% was priced.

Chart 7



There is no cash flow impact yet as the forward swap is just starting. The first Libor fixing takes place at a level of 3.30%. The client will receive a payment of 3.30% + the spread of 0.64%, a total of 3.94% against paying 6-month Libor to be set at its level in October 1993.

October 1993

On 12 October 1993, 6-month Libor is fixed at 3.36%, as shown on the curve above starting between Oct-93 and Oct-94. The client receives a payment for the first period based on the spread of 0.58% [3.94% 3.36%=0.58%].

The mark-to-market value of the LIA swap has moved slightly further positive, because the curve has continued to flatten. The position now has a MMV of 0.61%.

Table 7

Swap Component	PV
Floating Leg	9.9595%
Floating Leg Spread	1.5214%
EIA Floating Leg	-10.8687%
Swap	0.6123%

Subsequent Periods

For subsequent periods, the performance of the swap is summarized in the table below.

Table 8

Date	Libor	Spread	LIA	Net CF	MMV
12 Apr 1993					0.42%
12 Oct 1993	3.30%	0.64%	-3.36%	0.57%	0.61%
12 Apr 1994	3.36%	0.64%	-4.30%	-0.30%	-0.10%
12 Oct 1994	4.30%	0.64%	-5.76%	-0.82%	-0.13%
12 Apr 1995	5.76%	0.64%	-6.28%	0.11%	0.23%
12 Oct 1995	6.28%	0.64%	-5.77%	1.15%	0.31%
12 Apr 1996	5.77%	0.64%	-5.51%	0.90%	0.00%

The swap generates negative cash flows during 1994 after the Fed's rate rises of 0.25% in February, March and April, 0.50% in May and August, 0.75% in November and another 0.50% in February 1995.

Cash flow turns positive again in April 1995 because Libor is only 0.52% higher than it was the previous October and the client is receiving a spread of 0.64%. In October 1995 and April 1996 cash flow is again strongly positive because monetary policy has again been eased and short term rates have moved slightly lower.

Despite rate rises which destroyed bond markets and rendered huge losses on fixed income portfolios and other types of structured derivatives, the Libor-in-arrears swap has performed fairly well. In terms of cash flows, we can compare the internal rate of return on a loan with interest paid at Libor flat versus the same loan "hedged" with a LIA swap. Over the relevant 3-year period, the LIA hedge would have reduced borrowing costs by 0.24%:

Table 9

Date	Libor	LIA
12 Apr 1993	-100.00%	-100.00%
12 Oct 1993	3.30%	2.73%
12 Apr 1994	3.36%	3.66%
12 Oct 1994	4.30%	5.12%
12 Apr 1995	5.76%	5.64%
12 Oct 1995	6.28%	5.13%
12 Apr 1996	105.77%	104.87%
IRR	4.71%	4.47%

Despite very rapid rate rises, both of these floating-rate alternatives would have been better than locking into a fixed rate for the 3-year period 6 months forward. The relevant swap rate in October 1992 for the 3 years beginning April 1993 was 5.04% (expressed on the same semi-annual money market basis).

This example shows what makes LIA an attractive structure for borrowers. On numerous occasions during the past seven or eight years the forward curve has been far steeper than subsequent rate increases. Even during the devastating rises that took place in 1994, a LIA swap put in place in late 1992 showed attractive results.

Valuation Logic

The valuation of LIA swaps is fairly straightforward, with one complex twist that represents only a small part of the price.

Pricing a LIA swap is similar to pricing a regular interest rate swap, except that the mark-to-market values for the forward Libor rates are taken from the period beginning at each payment date, rather than ending at each payment date.

To price a 3-year EIA swap, for example, we use the strip of 6-month Euribor forwards beginning with the 6×12 FRA, but applying it to the 0×6 period.

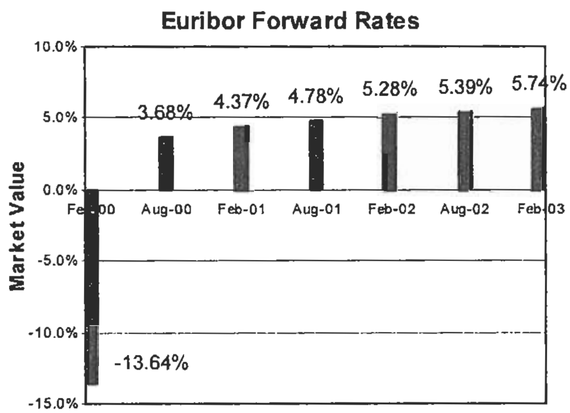
The following table shows 6-month Euribor forward rates in February 2000.

Table 10

Date	Euribor	Accrual	PVf	CF PV
02 Feb 2000			1.000000	13.6421%
02 Aug 2000	3.68%	0.5056	0.981735	1.8265%
02 Feb 2001	4.37%	0.5111	0.960266	2.1470%
02 Aug 2001	4.78%	0.5028	0.937752	2.2513%
04 Feb 2002	5.28%	0.5167	0.912852	2.4900%
02 Aug 2002	5.39%	0.4972	0.889032	2.3820%
03 Feb 2003	5.74%	0.5139	0.863579	2.5453%
03 Feb 2003	5.85%	0.5056	0.838784	

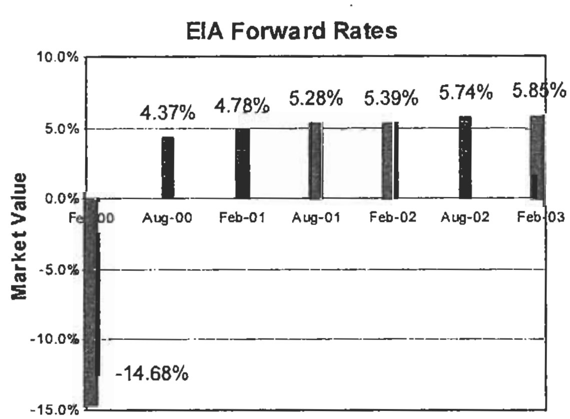
The normal Euribor strip over the next 3 years is worth 13.64% today. The following picture shows the forward rates in a "normal" 3-year interest rate swap.

Chart 8



If we shift them all forward by one period, we are pricing Euribor resets "in arrears." By doing this we incorporate the steepness of the yield curve.

Chart 9



The present value of the EIA resets is 14.68%. In effect, a slightly larger EIA reset replaces each "normal" forward rate. The PV of the EIA strip is thus bigger than the PV of a "normal" forward strip.

The EIA strip is priced as follows:

Table 11

Date	EIA	Accrual	PVf	CF PV
02 Feb 2000			1.000000	14.6756%
02 Aug 2000	4.37%	0.5056	0.981735	2.1711%
02 Feb 2001	4.78%	0.5111	0.960266	2.3436%
02 Aug 2001	5.28%	0.5028	0.937752	2.4892%
04 Feb 2002	5.39%	0.5167	0.912852	2.5414%
02 Aug 2002	5.74%	0.4972	0.889032	2.5353%
03 Feb 2003	5.85%	0.5139	0.863579	2.5949%

When we compare the PVs of the two strips, we can calculate the PV of the LIA spread:

Table 12

	PV	Spread
Euribor	13.64%	4.84%
EIA	14.68%	5.21%
Difference	1.03%	0.37%

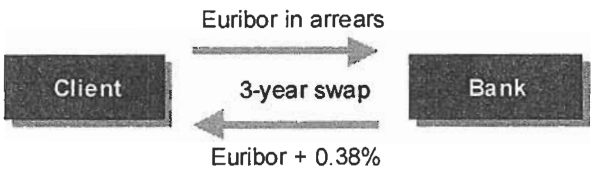
The spread we quoted above for this swap was 0.38%. Now we have a spread of 0.37%. The difference is due to the use of the Euribor resets in periods to which they do not naturally apply. We will explore this in the next section.

Embedded Optionality

Hedging a LIA Swap

Imagine you work on the swap desk that is taking the bank side of the client deal we showed above.

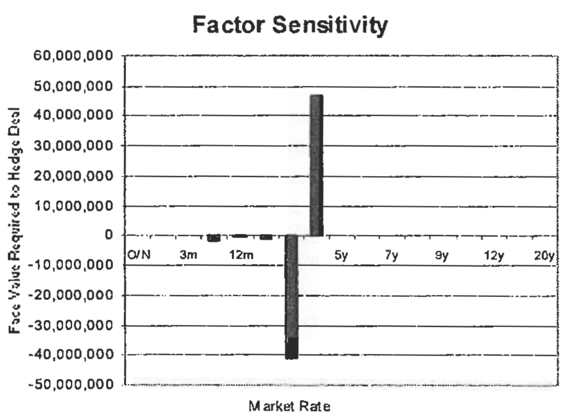
Euribor-in-Arrears Swap



Notional principal: €100 million

To hedge this position you will enter two offsetting swaps, as discussed above. Because you are paying Euribor, you will need to receive Euribor and pay fixed in a 3-year swap. Because you are receiving Euribor in arrears you will need to pay Euribor and receive fixed in a 4-year swap.

Chart 10



For the sake of illustration, let us assume that you enter into all five trades shown in the above chart in the amounts shown. Your hedges can be summarized as follows.

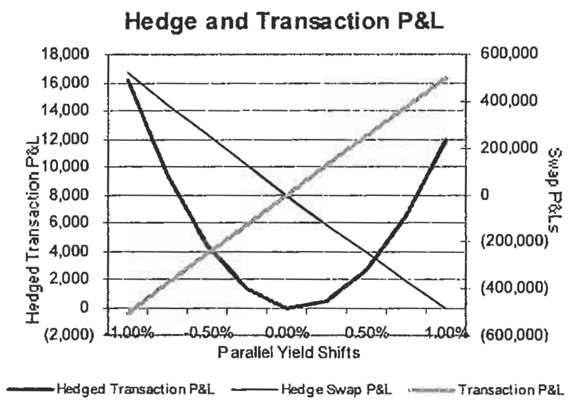
Term	Hedge Amount
6m	-1,415,393
12m	-440,985
2y	-1,080,925
3y	-41,015,919
4y	46,609,050

Positive values mean you receive fixed and negative values mean you pay fixed.

Notice that the face value required to hedge notional principal of €100 million is less than half this amount for each swap. This is because we will unwind the 4-year swap when it still has one year left. We will use its final two payments to hedge a single EIA payment, a ratio of 2 to 1. This is a rough idea for why the hedge is only half the notional principal of the client swap.

We now subject the hedged portfolio to parallel rate movements and measure what happens to its mark-to market value. Our goal is to verify how effective the hedge is. The results are shown in the chart following.

Chart 11

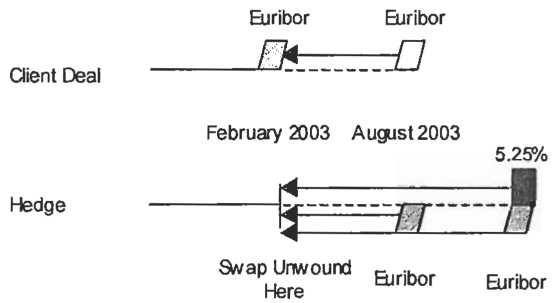


The hedge appears to be amazingly effective! It only makes money for your book. It cannot lose money. The only bad thing would be no rate movement at all, as your MMV value would stay at 0.

The two swaps you entered into to hedge the EIA swap transaction basically offset it. No matter which way rates move, however, you show residual profit.

The explanation for this wonderful state of affairs lies in the use of the 4-year swap to hedge the EIA receipts. The swap generates cash flows on the usual dates, going out to 4 years. Your client deal, however, has its last cash flow at Year 3. In truth, each of the hedge swap payments is to be paid 12 months after it is needed to hedge the client EIA deal.

For example, look at the EIA cash flow due to be set and paid at Year 3, the last payment date of the swap. The client is paying EIA to the bank. The bank's hedge is to pay Euribor to the market for the next two periods:



The principal difference in the client swap and the hedge swap lies in the value of any change in the 6-month Euribor fixing taken from the market on 2 February 2003. The hedge is based on a forward rate of 5.85%. Each single basis point of change in this rate results in a cash flow change for the last fixing of the EIA swap of 0.01% times the accrual factor for the 6 months ending February 2003.

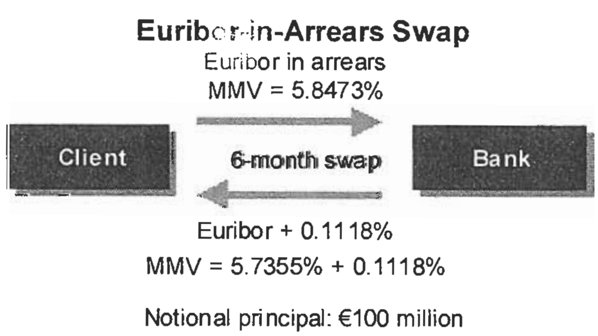
The difference in the hedge is that we have to ***unwind*** it, so the change in the payment is equal to the ***discounted value*** of the two Euribor fixings and the 5.25% fixed coupon.

Single-Period EIA Swap

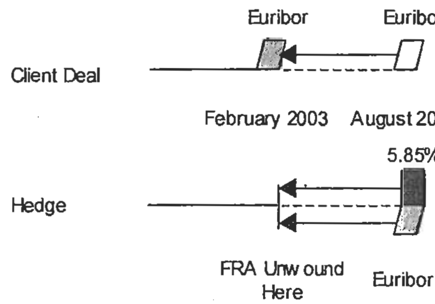
The explanation can be simplified if we imagine using forward rate agreements to lock in the value of each Euribor fixing, instead of offsetting swaps. The valuation and hedge logic are identical.

Imagine we price a single-period EIA swap for the six months running from August 2002 through February 2003. On February 2003 we will receive EIA and pay Euribor, which will have been set in August 2002.

The forward rates for the Euribor fixing and the EIA indicate a swap as follows:



The hedge of the final EIA payment is thus a single FRA for the six months running from February through August 2003. We can draw this similarly to the hedge above.



The hedge allows us to fix the value of the EIA payment from the client at 5.85%, the forward rate for this 6-month period at the beginning of the swap. However, we still have a ***mismatch in terms of cash flow value dates***. The hedge cash flow is required on 3 February 2003, not 4 August 2003. We are certain of the amount of cash we are to receive in August, but not its discounted value. This will change as the forward rate for the period changes.

In order to use a FRA to hedge the EIA cash flow, we have to calculate a hedge ratio. The PV01 for both positions is shown below.

Table 13

FRA Fixed	Euribor	Accrual	6-mo. PVf	CF PV
5.85%	5.86%	0.5056	0.971240	-0.0049%
5.85%	5.85%	0.5056	0.971288	0.0000%
5.85%	5.84%	0.5056	0.971335	0.0049%
Euribor + Spread	**Euribor**	**Accrual**	**6-mo. PVf**	**CF PV**
-5.85%	5.86%	0.5139	1.000000	0.0051%
-5.85%	5.85%	0.5139	1.000000	0.0000%
-5.85%	5.84%	0.5139	1.000000	-0.0051%

The FRA shows P&L of 0.0049% for a 0.01% change in Euribor. The same Euribor change affects the EIA flow only as much as the accrual factor, so the equivalent P&L is 0.005139%.

Notice that the accrual factor is different for the two legs. This is because we are using the accrual factor for the natural Euribor fixing period in the FRA, i.e. from February to August 2003, whereas the same Euribor

rate is applied to the prior period in the client swap, i.e. from August 2002 to February 2003.

The cash flow PV (***CFPV*** above) is calculated by discounting the Euribor flow back to the beginning of the 6-month period. The discount factor varies as Euribor changes. The formula for the CFPV when Euribor rises to 5.86% is:

$$\mathrm{CFPV}_{\mathrm{RateUp}} = \frac{(5.85\% - 5.86\%) \times 0.5056}{1 + 5.86\% \times 0.5056} = -0.0049\%$$

The cash flow for the EIA swap is simply the Euribor fixing times the accrual factor:

$$\mathrm{CFPV}_{\mathrm{RateUp}} = (5.86\% - 5.85\%) \times 0.5139 = 0.0051\%$$

We can summarize these results. The relative PV01 of the two positions is also shown labeled "Ratio".

Table 14

Euribor	EIA G/(L)	Hedge CF	Ratio
5.86%	0.005139%	-0.004910%	-104.658%
5.85%		**Average:**	**-104.653%**
5.84%	-0.005139%	0.004911%	-104.648%

When Euribor rises to 5.86%, the change in value of the client swap EIA cash flow is 0.005139%. The change in value of the FRA used to hedge the EIA payment varies slightly and averages 0.0049105%.

The ratio of the gains and losses, on average, is:

$$\text{Hedge Ratio} = \frac{0.005139\%}{0.0049105\%} = 104.653\%$$

Because the FRA hedge is discounted over the 6-month period, we need to use a small amount more of it to hedge the EIA cash flow.

Setting the hedge to this amount results in a balanced hedge:

Table 15

Euribor Δ	EIA G/(L)	Hedge CF	Net CF
0.01%	0.005139%	-0.005139%	0.000000%
-0.01%	-0.005139%	0.005139%	0.000000%

This is true for a change of 0.01% in Euribor. What happens when Euribor moves by a larger amount?

Table 16

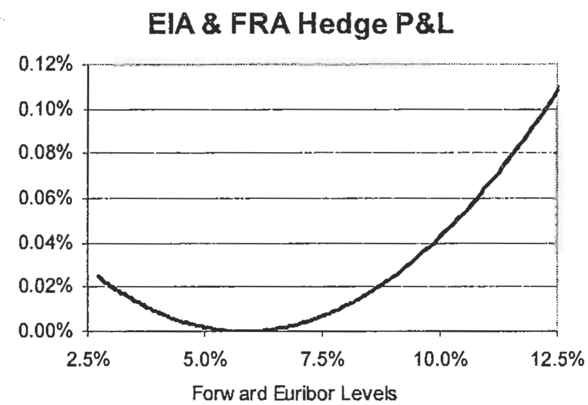
Euribor Δ	EIA G/(L)	Hedge CF	Net CF
0.10%	0.051389%	-0.051364%	0.000025%
-0.10%	-0.051389%	0.051414%	0.000025%

Table 17

Euribor Δ	EIA G/(L)	Hedge CF	Net CF
0.50%	0.256944%	-0.256315%	0.000629%
-0.50%	-0.256944%	0.257577%	0.000632%

In both cases the hedge performs well. In fact, as Euribor moves further and further, the hedge produces more and more profit.

Chart 12



This is because the change in value of the EIA cash flow is linear. As Euribor moves by a basis point, the cash flow moves by exactly 1 basis point times the accrual factor. This is as true for the first basis point as it is for the 50th.

The hedge, however, exhibits convexity normal for all fixed rate instruments. When it loses money it loses less than it gains when it gains money. The effect is always positive—if we are receiving the EIA payments. If we were paying EIA, the effect would be just the opposite.

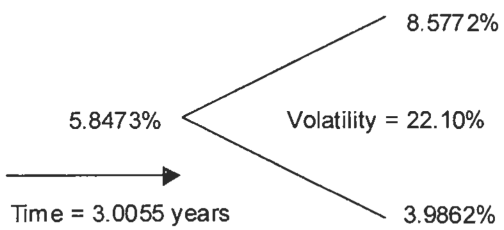
No one in the market who recognizes this will be willing to pay EIA at the level we have shown above. Everyone would be willing to receive it. In order for trades to take place, we have to ***adjust*** the Euribor forward rates. This is the ***convexity adjustment*** we referred to above.

To induce payers of EIA, the market will give them some of the receivers' riskless gains. The amount of gain rises as we move Euribor further and further. We have to stipulate an amount of movement we reasonably ***expect*** to see. This should be based on a market rate, which in this case is the caplet volatility for each respective Euribor fixing.

The mechanism for the EIA receiver to give up some of his riskless gain is to make him pay a slightly larger

spread to the EIA payer. To calculate a spread amount that is slightly larger, we need to use forward rates that are slightly higher. We have to adjust the forward rates so that the spread changes by the amount of riskless profit we expect.

Suppose market caplet volatility for the 6-month Euribor fixing on 3 February 2003 is 22.10%. This date lies 3.0055 years in the future from the value date of 2 February 2000 (because 2 February 2003 falls on a Sunday). Market volatility suggests the following range for Euribor at the fixing date:



The up and down values are calculated using volatility and time. Our underlying assumption is that the forward rates are distributed lognormally. This is the same as saying that the logs of the forward rates are distributed normally. We can adjust them up and down by the volatility, since it is taken to represent one standard deviation in the distribution of the logs of the forward rates. The relationship is:

$$\text{Euribor}_{\text{RateUp/Down}} = \exp\left[\ln(\text{FRA}) \pm \sigma\sqrt{t}\right]$$

For the upper forward rate:

$$\begin{aligned}\text{Euribor}_{\text{RateUp}} &= \exp\left[\ln(3.8473\%) + 22.10\% \times \sqrt{3.0055}\right] \\ &= 8.5772\%\end{aligned}$$

Our "expectation" for the value of the Euribor fixing on 3 February 2003 is 5.8473% with a range from 3.9862% up to 8.5572% within one standard deviation. If we increase the forward rate enough to bring the riskless profit to 0 at the two range boundaries, we can identify the middle of the market. Both the EIA payer and receiver should be indifferent at this adjusted rate, as both "expect" a P&L of 0.

On the upper range, the P&L from the hedge FRA and the client deal would be as shown below.

Euribor	EIA G/(L)	Hedge CF	Adj.
8.5772%	1.402883%	-1.384326%	-0.018557%

At a Euribor level of 8.5772%, the EIA swap with the client has positive P&L of 1.4029%. This is the difference between the one-period forward FRA and the Euribor fixing:

$$\begin{aligned}\text{EIA P\&L} &= (\text{Euribor} - \text{FRA}) \times \alpha \\ &= (8.5772\% - 5.8473\%) \times 0.5139 \\ &= 1.4029\%\end{aligned}$$

The FRA hedge has P&L of -1.3843%. This is the discounted value of the difference between Euribor and the FRA fixed rate adjusted by the slightly larger face value on the hedge:

$$\begin{aligned}\text{FRA P\&L} &= \frac{(\text{FRA} - \text{Euribor}) \times \alpha}{(1 + \text{FRA} \times \alpha)} \times \text{Notional Principal} \\ &= \frac{(5.8473\% - 8.5772\%) \times 0.5056}{(1 + 8.5772\% \times 0.5056)} \times 104.6532\% \\ &= -1.3843\%\end{aligned}$$

To balance these, we need the EIA receiver to pay out an amount equal to 0.0186%. This is the ***convexity adjustment*** against which he can expect to break even:

$$\begin{array}{r} 1.402883\% \\ -1.384326\% \\ \hline 0.018557\% \end{array}$$

This amount of cash adjustment, in turn, represents a rate adjustment of:

$$\frac{0.018557\%}{0.5139} = 0.036111\%$$

If we make the same calculation on the lower range boundary around the forward rate, we get a smaller amount:

Euribor	EIA G/(L)	Hedge CF	Adjustment
3.9862%	-0.956377%	0.965197%	-0.008820%

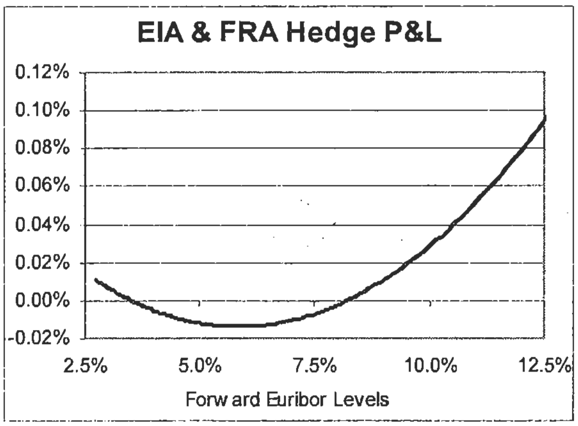
The cash adjustment is 0.008820%, which is a rate adjustment of 0.017164%.

The average of the two rate adjustments is:

$$\frac{0.036111\% + 0.017164\%}{2} = 0.026638\%$$

This is the amount by which we should increase the forward Euribor rate when pricing the deal. If we do, the hedged transaction P&L looks as follows:

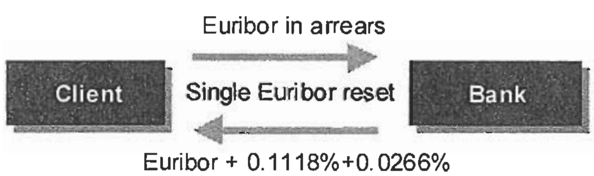
Chart 13



The P&L shown above has an "expected value" of zero. This means that if we take each of the possible levels of P&L shown and weight them by their risk-neutral probabilities, the weighted average is 0. There is an equal likelihood of making money and losing money.

The quote for this single-period EIA swap would therefore be:

Euribor-in-Arrears Swap



Analytic Convexity Adjustment Formula

The convexity adjustment can be calculated using caplet volatilities, which is explained by John Hull in *Options, Futures & Other Derivatives*, fourth edition, pages 550 and 553. Hull shows that the adjustment to the forward rates for calculating LIA is (using notation like that above):

$$\frac{FRA_n^2 \times \sigma_n^2 \times \alpha_n \times t_n}{1 + FRA_n \times \alpha_n}$$

where:

FRA_n =	Libor forward rate observed in the market on the value date for the reset period beginning at *n*
σ_n =	Lognormal volatility of the FRA_n
α_n =	Accrual factor for the reset period beginning at *n*
t_n =	Time from the value date to *n*

Using this formula to calculate the convexity adjustment for the period described above:

$$\frac{5.8473\%^2 \times 22.10\%^2 \times 0.5056 \times 3.0055}{1 + 5.8473\% \times 0.5056} = 0.0246\%$$

This measure is roughly equivalent to that calculated above.

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